Modeling and Hamiltonian control of radial transport in magnetized plasmas with linear configuration

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Seminar on results of my Ph.D. in Theoretical Physics and Mathematics
Ph.D. supervisors: Cristel CHANDRE & Guido CIRAOLI
Goal: Production of energy by fusion reactions

Fusion by different confinements
- gravitational (sun)
- inertial (laser: LMJ)
- magnetic (tokamak: Iter)

Magnetic confinement
- instabilities (ELM, $E \times B$, ...)
  - less confinement
  - material resistance

Objectives:
- understanding of instabilities
- understanding of plasma dynamics
- how to increase the confinement
Goal: Production of energy by fusion reactions

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Objectives:
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- how to increase the confinement
Introduction > Radial transport

- Plasma with linear configuration
  - less sources of instabilities
  - constant magnetic field $B = B(x, y)\hat{z}$
  - plasma less hot than in tokamak
  - measuring and modifications on plasma (probes)

VINETA = Versatile Instrument for studies on Nonlinearity, Electromagnetism, Turbulence and Applications.
Localization: IPP (Max-Planck-Institut für Plasma-Physik at Greifswald, Germany)
Title: Modeling and Hamiltonian control of radial transport in magnetized plasmas with linear configuration

- Modeling of plasma dynamics
  - Reduced models
  - Hamiltonian structure
- Barrier to the radial transport due to the $\mathbf{E} \times \mathbf{B}$ drift
  - Computation to observe radial transport
  - Algorithm for the control
  - Computation to observe the control of radial transport
- Study of the feedback
  - Effect of probes on auto-coherent model
The variables of a plasma

- composition of a plasma: $e^-$, $\text{Ar}^+$, ... (species $s$)
- variables = physical quantities describing plasma dynamics
- 2 main descriptions

<table>
<thead>
<tr>
<th>kinetic</th>
<th>fluid</th>
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<tbody>
<tr>
<td>$f_s(x, v, t)$ distribution function</td>
<td>$n_s(x, t)$ density of particles</td>
</tr>
<tr>
<td>$v_s(x, t)$ fluid velocity</td>
<td>$T_s(x, t)$ temperature</td>
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<td>...</td>
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</tbody>
</table>

\[ \mathbf{E}(x, t) = -\nabla \phi(x, t) \] electric field

\[ \mathbf{B} = B(x, y) \hat{z} \] external magnetic field (frozen)
### Introduction > Plasma dynamics

<table>
<thead>
<tr>
<th>advantages</th>
<th>kinetic</th>
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<td>exact for a non collisional plasma</td>
<td>adapted for measurements</td>
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<td>disadvantages</td>
<td>numerical cost &amp; physical interpretation</td>
<td>closure</td>
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<td>equations</td>
<td>Vlasov</td>
<td>moments of Vlasov</td>
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\[
\frac{\partial f_s}{\partial t} = -\mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} - \frac{e_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}}
\]

\[
\frac{\partial n_s}{\partial t} = -\nabla \cdot (n_s \mathbf{v}_s)
\]

\[
\frac{\partial \mathbf{v}_s}{\partial t} = -(\mathbf{v}_s \cdot \nabla)\mathbf{v}_s - \frac{\nabla \cdot \overline{P_s}}{m_s n_s} + \frac{e_s}{m_s} (\mathbf{E} + \mathbf{v}_s \times \mathbf{B})
\]

+ equations of fields $\mathbf{E}$ and $\mathbf{B}$

---

O. IZACARD (CPT)  
Modeling & control of radial transport  
Imperial: 15-02-2012  
6 / 30
2 species: \( n_i, n_e, \mathbf{v}_i \) et \( \mathbf{v}_e \)

Equations of conservation for each species \( s = \{e, i\} \)

\[
\frac{\partial n_s}{\partial t} = - \nabla \cdot (n_s \mathbf{v}_s)
\]

\[
\mathbf{v}_s = \frac{\mathbf{\hat{z}} \times \nabla \phi}{B} + \frac{\mathbf{\hat{z}} \times \nabla T_s \log n_s}{e_s B} - \frac{\mathbf{\hat{z}} \times \nabla \cdot \mathbf{\bar{\pi}}_s}{e_s} - \frac{\mu_s}{e_s B} \left( \frac{\partial}{\partial t} + \mathbf{v}_s \cdot \nabla \right) \mathbf{\hat{z}} \times \mathbf{v}_s
\]

- closure on pressure tensor: \( \mathbf{\bar{\pi}}_s = \mathbf{\bar{\pi}}_s(n_s, \mathbf{v}_s) \)
Introduction > Fluid models for 2 species

- 2 species: \( n_i, n_e, \mathbf{v}_i \) et \( \mathbf{v}_e \)
- Equations of conservation for each species \( s = \{ e, i \} \)

\[
\frac{\partial n_s}{\partial t} = -\nabla \cdot (n_s \mathbf{v}_s) \\
\mathbf{v}_s = \mathbf{v}_E \times \mathbf{B} + \mathbf{v}_{\text{dia},s} + \mathbf{v}_{\text{pol},s} + \mathbf{v}_{\text{FLR}}
\]

- closure on pressure tensor: \( \bar{\pi}_s = \bar{\pi}_s(n_s, \mathbf{v}_s) \)
- quasi-neutrality: \( n_i = n_e = n \ (\lambda_D \ll L) \)
Introduction > Fluid models for 2 species

- 2 species: $n_i$, $n_e$, $\nu_i$ et $\nu_e$

Equations of conservation for each species $s = \{e, i\}$

$$\frac{\partial n_s}{\partial t} = -\nabla \cdot (n_s \nu_s)$$

$$\nu_s = \nu_E \times B + \nu_{\text{dia},s} + \nu_{\text{pol},s} + \nu_{\text{FLR}}$$

- closure on pressure tensor: $\overline{\pi}_s = \overline{\pi}_s(n_s, \nu_s)$

- quasi-neutrality: $n_i = n_e = n \ (\lambda_D \ll L)$

- equations given by:
  - $\frac{\partial n}{\partial t} = -\nabla \cdot (n\nu_e)$ for electrons
  - $\nabla \cdot j = 0$, where $j = en(\nu_i - \nu_e)$
  - with the ordering on velocities
Hypothesis

- Maxwellian (thermodynamic equilibrium): $\bar{\pi}_i = \bar{\pi}_e = 0$
- cold ions: $T_i = 0$ and $T_e = 1$
- constant and homogeneous magnetic field $\mathbf{B} = \hat{z}$
- ordering on fluctuations: $n = 1 + \varepsilon \tilde{n}, \phi = \varepsilon \tilde{\phi}$ ($\varepsilon \ll 1$)
- polarization velocity for electrons neglected in front of ones for ions

\[ \mathbf{v}_e = \varepsilon \mathbf{v}_e^{(1)} = \varepsilon \mathbf{v}_{E \times B} + \varepsilon \mathbf{v}_{\text{dia},e} = \varepsilon \hat{z} \times \nabla \phi - \varepsilon \hat{z} \times \nabla n \]

\[ \mathbf{v}_i = \varepsilon \mathbf{v}_{E \times B} + \varepsilon^2 \mathbf{v}_{\text{pol},i} \]

\[ \mathbf{v}_{E \times B} = \hat{z} \times \nabla \phi \]

\[ \mathbf{v}_{\text{pol},i} = \left( \frac{\partial}{\partial t} - (\hat{z} \times \nabla \phi) \cdot \nabla \right) \nabla \phi \]

Reduced model

\[ \frac{\partial n}{\partial t} = - [\phi, n] \]

\[ \frac{\partial \Delta \phi}{\partial t} = - [\phi, \Delta \phi] \]

with the canonical bracket $[f, g] = \hat{z} \cdot \nabla f \times \nabla g$
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Add of the temperature of ions and magnetic inhomogeneity

Transport barrier

Probes in an auto-coherent model

\[-[\phi, n] = \text{electrical drift instability}\]
\[\left[n, \frac{1}{B}\right] = \text{interchange instability} \ (\sim \text{Rayleigh-Taylor in hydrodynamics})\]
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- Add of the temperature of ions and magnetic inhomogeneity
- **Transport barrier**
- **Probes in an auto-coherent model**

$- [\phi, n] = \text{electrical drift instability}$

$[n, \frac{1}{B}] = \text{interchange instability (\sim Rayleigh-Taylor in hydrodynamics)}$
1. Gyromap > Braginskii & ordering

- Closure by Braginskii’s tensor (near thermodynamics equilibrium, friction terms...)

\[
v_{\text{FLR}} = -\frac{\alpha_s T_s}{n e_s B} \hat{z} \times \nabla (n \nabla \cdot v_s)
\]
\[
- \frac{\mu_s T_s}{e_s 2B} \left[ \Delta v_s + (\hat{z} \times \nabla \log n \cdot \nabla)(\hat{z} \times v_s) + (\nabla \log n \cdot \nabla) v_s \right]
\]

- Ordering on fluctuations (\(\varepsilon \ll 1\))

\[
1/B = 1 + \varepsilon/\tilde{B},
\]
\[
\phi = \varepsilon \tilde{\phi},
\]
\[
n = 1 + \varepsilon \tilde{n},
\]
\[
v_s = \varepsilon v_s^{(1)} + \varepsilon^2 v_s^{(2)}.
\]

- FLR effects appear at 1\(^{\text{st}}\) order

\[
\left( e_s + \mu_s \frac{T_s}{2} \Delta \right) v_s^{(1)} = \hat{z} \times \nabla (e_s \phi + T_s n),
\]

- \(T_e = 1\) et \(T_i = T\)
1. Gyromap > Reduced fluid model

With the natural change of variables \[ (1 + \frac{T}{2} \Delta) \Phi = \phi + Tn \] we found the ion velocity at the 1\textsuperscript{st} order

\[ v_i^{(1)} = \hat{z} \times \nabla \Phi \]

Equations of conservation

\[
\frac{\partial n}{\partial t} = - \left[ \Phi + \frac{T}{2} \Delta \Phi, n + \frac{1}{B} \right] + (1 + T) \left[ n, \frac{1}{B} \right] \\
\frac{\partial \Delta \Phi}{\partial t} = - [\Phi, \Delta \Phi] + (1 + T) \left( 1 + \frac{T}{2} \Delta \right) \left[ n, \frac{1}{B} \right] \\
+ T \nabla \cdot \left[ n + \frac{1}{B}, \nabla \Phi \right] - \frac{T^2}{4} \Delta \left[ \Delta \Phi, n + \frac{1}{B} \right]
\]
1. Gyromap > Hamiltonian description

- Hamiltonian description

\[ \frac{\partial n}{\partial t} = \{n, H\} \]
\[ \frac{\partial \Delta \Phi}{\partial t} = \{\Delta \Phi, H\} \]

\[ H(n, \Delta \Phi) = \frac{1}{2} \int d^2x \left( (1 + T)n^2 + |\nabla \Phi|^2 \right) \]

\[ \{F, G\} = \int d^2x \left( \left( n + \frac{1}{B} \right) \left( \frac{\delta F}{\delta n} + \frac{T}{2} \Delta \frac{\delta F}{\delta \Delta \Phi} \right) \frac{\delta G}{\delta n} \right) \]
\[ + \left( n + \frac{1}{B} \right) \left[ \frac{\delta F}{\delta n} + \frac{T}{2} \Delta \frac{\delta F}{\delta \Delta \Phi} \right] \frac{\delta G}{\delta n} \]
\[ + \left( \Delta \Phi - \frac{T}{2} \Delta n \right) \left[ \frac{\delta F}{\delta \Delta \Phi}, \frac{\delta G}{\delta \Delta \Phi} \right] \]

- Poisson bracket (bi-linearity, anti-symmetry, Leibniz’s rule and Jacobi identity)
1. Gyromap > Hamiltonian description

- Hamiltonian description

\[ \frac{\partial n}{\partial t} = \{ n, H \} \]
\[ \frac{\partial \Delta \Phi}{\partial t} = \{ \Delta \Phi, H \} \]

- Advantages
  - no fake dissipations (no “mutilations”)
  - accessibility to Casimirs (invariant families)
    \( \Rightarrow \) verification of numerical codes
    \( \Rightarrow \) equilibrium stability
  - possibility to use the perturbation theory
    \( \Rightarrow \) control of the dynamics
1. Gyromap > The gyromap

- closure by Braginskii & inhomogeneity $B$
- Hamiltonian structure known without ion temperature
  \[
  \frac{\partial n}{\partial t} = -[\phi, n] - [\phi, \frac{1}{B}] + [n, \frac{1}{B}]
  \]
  \[
  \frac{\partial \Delta \phi}{\partial t} = -[\phi, \Delta \phi] + [n, \frac{1}{B}]
  \]
- add of the ion temperature by the Hamiltonian (add of FLR effects)
  \[
  H(n, \Phi) = \frac{1}{2} \int d^2x \left( (1 + T)n^2 + |\nabla \Phi|^2 \right)
  \]
  with gyromap : $(N, \xi) \rightarrow (n, \Delta \Phi - \frac{T}{2} \Delta n)$ with
  \[
  \left(1 + \frac{T}{2} \Delta \right) \Phi = \phi + Tn.
  \]
- conservation of Hamiltonian structure
1. Gyromap > The gyromap

- closure by Braginskii & inhomogeneity $\mathbf{B}$
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  \]
- conservation of Hamiltonian structure

**O. Izacard, C. Chandre, E. Tassi, G. Ciraolo, Physics of Plasmas 18 (2011) 062105**

Gyromap for a two-dimensional Hamiltonian fluid model derived from Braginskii’s closure for magnetized plasmas
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Add of the temperature of ions and magnetic inhomogeneity

Transport barrier

Probes in an auto-coherent model
2. Control > Use of tracers

- \( \frac{\partial n}{\partial t} = - [\phi, n] = \frac{\partial n}{\partial x} \frac{\partial \phi}{\partial y} - \frac{\partial n}{\partial y} \frac{\partial \phi}{\partial x} \): linear ordinary with partial derivatives equation of 1\(^{st}\) order

- Method of characteristics
  
  \( n(s) = n(x(s), y(s), s) \) where \( n \) est constant along the characteristics \((x(s), y(s))\) \((dn(s)/ds = \dot{n} = 0)\)
  
  Hence
  
  \[
  \frac{dn(s)}{ds} = \frac{\partial n}{\partial t} + \frac{\partial n}{\partial x} \dot{x} + \frac{\partial n}{\partial y} \dot{y}
  \]

- solutions for the dynamical equations of tracers

  \[
  \begin{align*}
  \dot{x} &= \frac{dx}{dt} = -\frac{\partial \phi}{\partial y} \\
  \dot{y} &= \frac{dy}{dt} = \frac{\partial \phi}{\partial x} \\
  \dot{r} &= \frac{dr}{dt} = -\frac{1}{r} \frac{\partial \phi}{\partial \theta} \\
  \dot{\theta} &= \frac{d\theta}{dt} = \frac{1}{r} \frac{\partial \phi}{\partial r}
  \end{align*}
  \]
2. Control \rightarrow Static electric potential

- if $\phi = \phi(x, y)$ is an invariant in time:
  - tracers follow iso-potential curves
2. Control > Choice of an electric potential

- Dynamical potential
2. Control > Choice of an electric potential

- Dynamical potential

  Initial conditions

  ⇒ Radial transport

- Chaotic dynamics and radial transport
2. Control > Control by forcing

- intuitive method $\phi = 0$ on the circle

- $\Rightarrow$ Transport barrier

- Problem: energetic cost
2. Control > Algorithm of Hamiltonian control

- Dynamical equations

\[
\begin{align*}
\dot{r} &= -\frac{1}{r} \frac{\partial \phi_c}{\partial \theta} \\
\dot{\theta} &= \frac{1}{r} \frac{\partial \phi_c}{\partial r}
\end{align*}
\]

- The control is solution of:

\[
\begin{align*}
 r &= R(\theta, t) \\
\phi_c(r, \theta, t) &= \phi(f(r, \theta, t), \theta, t)
\end{align*}
\]

We choose: \( f(R, \theta, t) = r_0 \)
position of probes of measurement

\[
\begin{align*}
\phi_c(r, \theta, t) &= \phi(r + r_0 - \sqrt{r_0^2 - 2\Gamma \partial_\theta \phi(r_0, \theta, t)}, \theta, t) \\
R(\theta, t) &= \sqrt{r_0^2 - 2\Gamma \partial_\theta \phi(r_0, \theta, t)}, \Gamma = \text{primitive in } t
\end{align*}
\]
2. Control > The potentials

- Uncontrolled vs controlled potentials

Initial electric potential

Controlled electric potential
2. Control > Numerical results

- Numerical results for test particles

Poincaré section without control

Poincaré section with control
2. Control > The establishment of control

- goal
  - study of the robustness of the control
  - establishment of control with experimental constraints
- measurement and perturbation method
  - Langmuir probes

Langmuir probes of VINETA

Characteristics curve
2. Control > Langmuir probes

- Application of control by Langmuir probes

Parameters: 32 probes at $r_0 = \pi$, exponential decrease...

Ideal modification

Modification by probes
2. Control > Langmuir probes

- Application of control by Langmuir probes

![Graph showing confinement percentage over time with different control methods.]

- with ideal control
- with control injected by 32 probes
- with control injected par 32 probes and measurements by $5 \times 128$ probes
- without control
2. Control > Langmuir probes

- Application of control by Langmuir probes

With ideal control
- With control injected by 32 probes
- With control injected par 32 probes and measurements by 5 × 128 probes
- Without control

O. Izacard, N. Tronko, C. Chandre, G. Ciraolo, M. Vittot, Ph. Ghendrih
Plasma Physics and Controlled Fusion **53** (2011) 125008

Transport barrier for the radial diffusion due to the $E \times B$ drift motion of guiding centers in cylindrical confinement geometry
2. Control > Control on the fluid density

- control of tracers = control of the density

\[ N(t) = \int_0^{2\pi} \int_0^R n(r, \theta, t) r dr d\theta \text{ is constant} \]

\[ \Leftrightarrow \frac{\partial R}{\partial t} + \frac{1}{r} \frac{\partial}{\partial \theta} (\phi_c(R(\theta, t), \theta, t)) = 0 \]

⇒ same solution than the control on tracers
### 3. Feedback

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- **Add of the temperature of ions and magnetic inhomogeneity**
- **Transport barrier**
- **Probes in an auto-coherent model**
Fluid model for the edge of tokamak (scrape-off-layer SOL):

\[
\begin{align*}
\frac{\partial n}{\partial t} &= -[\phi, n] + D\Delta n + \sigma_\parallel e^{\Lambda - \phi} + S \\
\frac{\partial \Delta \phi}{\partial t} &= -[\phi, \Delta \phi] + \left[ n, \frac{1}{B} \right] + \sigma_\parallel (1 - e^{\Lambda - \phi}) + \mu \Delta^2 \phi
\end{align*}
\]

avec : \([f, g] = \frac{B}{B} \cdot (\nabla f \times \nabla g)\)

\[\left[ n, \frac{1}{B} \right] = \text{interchange, magnetic field depend only of variable } x, \text{ i.e. } 1/B = (R_0 + x)/(R_0 B_0)\]
3. Feedback > Radial transport

- TOKAM-2D: observation of turbulent radial transport
3. Feedback > Effect of a forcing by probes

- effect of the control by a forcing by Langmuir probes

\[
\begin{align*}
\frac{\partial n}{\partial t} &= -[\phi, n] + D\Delta n + \sigma_|| e^{\Lambda+\phi_{bias}} - \phi + S \\
\frac{\partial \Delta \phi}{\partial t} &= -[\phi, \Delta \phi] + \left[n, \frac{1}{B}\right] + \sigma_|| \left(1 - e^{\Lambda+\phi_{bias}} - \phi\right) + \mu \Delta^2 \phi
\end{align*}
\]

⇒ auto-coherent model
⇒ parallel current can be modified by Langmuir probes
3. Feedback > Conclusion & perspectives

Conclusion
- feedback of the density on the potential
- feasibility of perturbations by Langmuir probes
- control by forcing (high energetic cost)
- difficulties of a control by perturbations (temporal mean of potential no null, fluctuations of the position of the barrier too high...)

Perspectives
- intermittent control
- control in some regions
- other theoretical solution
## Conclusion & Resume

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Add of the temperature of ions and magnetic inhomogeneity

Temperature of ions added with Hamiltonian structure

Algorithm, Robustness of control